# A Chirp-Based Frequency Synchronization Approach for Flat Fading Channels


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***Abstract*—In different new and emerging technologies, where high frequency offsets (FOs) are expected due to the low-cost nature of the local oscillators, and low sampling rates are chosen for the sake of power efficiency, accurate FO estimation becomes a challenging task. This work proposes a frequency synchronization approach based on a dual-chirp training sequence. Its performance is evaluated by means of simulations and validated with the derived Cramer-Rao lower bound. It is shown that the proposed method achieves near-optimum performance with lower complexity than the state-of-the-art approach.**



***Index Terms*—Frequency offset estimation, synchronization, up-down-chirp, dual-chirp, reference sequences, CRLB.**


## I. Introduction

The cost-effectiveness sought by different new and emerging technologies entails the choice of low-cost local oscillators. Therefore, due to their low accuracy, high frequency offsets (FOs) may appear between the reference frequency at the receiver used for down-conversion and the carrier frequency of the received signal. Moreover, in order to achieve low complexity at the end device, a low sampling rate for initial synchronization is commonly selected. Thus, the low accuracy associated to a low sampling rate, together with the potentially high FOs, poses an important challenge in the FO estimation, a crucial task that has to be appropriately accomplished to avoid performance degradation at the receiver.

Aiming at high probability of detection in the low signal-to-noise ratio (SNR) regime and moderate latency, a detection approach based on matched filtering is widely used in the literature. The robustness of chirp sequences against high FOs in a matched filter-based detection, as well as their special time and frequency properties, have motivated their application in the detection process and in the estimation of the synchronization parameters [1] [2]. In [1], a training sequence consisting of a linear up-chirp followed by its conjugate, known as linear up-down-chirp, assists the synchronization process under additive white Gaussian noise (AWGN) channel conditions. For the sake of low latency, fractional and integer frequency offsets are estimated independently. The periodicity obtained after downsampling a linear chirp is utilized for the estimation of the fractional frequency offset (FFO) with an approach based on autocorrelation (AC). For the calculation of the integer frequency offset (IFO), a laborious algorithm is implemented, which requires the center value above a certain threshold at the output of each matched filter (MF).

A dual-chirp, which consists of a linear up-chirp transmitted simultaneously with its complex conjugate down-chirp, acts as a reference signal for synchronization in AWGN channels in [3]. The outputs of the MFs matched to each of the linear chirps are exploited for the estimation of the FO in frequency domain. However, its performance is linked to a high computational complexity due to the calculation of a discrete Fourier transform (DFT) with an upsampling factor and a subsequent polynomial interpolation.

This work proposes a frequency synchronization approach with a dual-chirp as a reference signal, suitable for flat fading channels under high FOs. Unlike in [3], the FO estimation is accomplished in time domain. The FFO, obtained through AC, is compensated in the reference signal prior to the IFO estimation. The IFO can thus be easily estimated using the positions of the global maxima at the outputs of the MFs.

Under the assumption of perfect time synchronization, the proposed approach is derived for a dual-chirp and for a linear up-down-chirp. Theoretical limits in terms of the Cramer Rao lower bounds (CRLBs) are derived for each reference sequence, which serve as benchmarks for results evaluation. Comprehensive simulations have been conducted to verify the validity of the presented technique. For the purpose of comparisons, the DFT-based approach described in [3] has been implemented with different upsampling factors and evaluated with both reference sequences. It is shown that the proposed approach achieves a performance close to the CRLB with low computational complexity, and significantly outperforms the state-of-the-art algorithm for moderate to high SNR regimes.

The remainder of this paper is structured as follows. In Section II, the system model and the reference sequences are introduced. The proposed synchronization algorithm is presented in Section III. In Section IV, the CRLBs are derived. Simulation results are provided and discussed in Section V. Finally, conclusions are drawn in Section VI.

## II. System Model

Under perfect time synchronization, the received baseband signal in discrete time domain can be described as

$$r[n] = hx[n]e^{j2\pi\epsilon n} + w[n], \qquad (1)$$

where $h$ represents the complex channel gain, unknown but constant over the transmission duration of the reference signal $x[n]$, $\epsilon$ indicates the FO, normalized by the sampling frequency $f_s$, and $w[n]$ denotes complex AWGN with zero mean and variance $\sigma^2$.

### A. Linear Up- and Down-Chirp Signals

The linear up- and down-chirp sequences can be expressed as

$$x_u[n] = e^{j\pi\left(\frac{n^2}{N_l}-n\right)}, \quad \text{and} \quad x_d[n] = e^{-j\pi\left(\frac{n^2}{N_l}-n\right)}, \tag{2}$$

where $n = 0, 1, ..., N_l - 1$. The length of the sequence, $N_l$, known as compression factor in radar systems, is given by the time-bandwidth product $N_l = BT_l$, where $B$ is the frequency band swept during the time span $T_l$. The reference sequence built with a linear up-chirp followed by its complex conjugate, or down-chirp, is thus given by

$$x_l[n] = \begin{cases} x_u[n], & n = 0, 1, ..., N_l - 1, \\ x_u^*[n - N_l], & n = N_l, N_l + 1, ..., 2N_l - 1. \end{cases} \tag{3}$$

### B. Dual-Chirp Signal

The dual-chirp, also referred to as composite chirp, consists of one linear up-chirp and its corresponding down-chirp, which are transmitted simultaneously. Since the down-chirp is the complex conjugate of the up-chirp, the resulting signal is purely real-valued, and can be formulated as

$$x_c[n] = \alpha \cos\left(\pi\left(\frac{n^2}{N_c} - n\right)\right), \tag{4}$$

where $n = 0, 1, ..., N_c - 1$, being $N_c$ the length in samples of the dual-chirp, and $\alpha$ represents a normalization factor to ensure unitary power.

In terms of the underlying linear chirps, $x_c[n]$ can be expressed as

$$x_c[n] = \frac{\alpha}{2}\left(x_u[n] + x_d[n]\right), \tag{5}$$

being $x_u[n]$ and $x_d[n]$ the sequences given in (2), of length $N_c$.

## III. Frequency Offset Estimation

The estimation of the FO is accomplished in different stages. The FFO is initially calculated and subsequently compensated in the received reference sequence. Afterwards, using the sequence with compensated FFO, the IFO is found. Without loss of generality, the detailed analysis presented in the next subsections is based on the up-down-chirp, and thereafter the modifications to be done when applying the dual-chirp are explained.

### A. Fractional Frequency Offset Estimation

The method employed for the estimation of the FFO takes advantage of the periodicity generated after downsampling a chirp sequence. In particular, downsampling a linear chirp with compression factor $N_i$ by 2 results in another linear chirp sequence with compression factor $N_i/4$. Thus, $N_i$ should be an integer multiple of 4. The downsampled sequence exhibits a periodicity of $N_i/4$ samples that can be exploited for synchronization with an AC-based approach. Specifically, the phase of the AC of a periodic signal at its maximum provides an estimation of the FFO [4].

At the entrance of the synchronization block, a demultiplexer extracts samples from the received signal $r[n]$ alternatively, generating two downsampled signals, $r_1[n] = r[2n]$, and $r_2[n] = r[2n + 1]$.

Considering the linear up-down-chirp as reference, the first $N_l/2$ samples of each demultiplexed sequence correspond to a downsampled version of the received up-chirp, whilst the second $N_l/2$ samples refer to the down-chirp as:

$$\begin{aligned} r_{u,1}[n] &= r_1[n], & r_{d,1}[n] &= r_1[n + N_l/2], \\ r_{u,2}[n] &= r_2[n], & r_{d,2}[n] &= r_2[n + N_l/2], \end{aligned} \tag{6}$$

for $n = 0, 1, ..., N_l/2 - 1$. Consequently, four different signals of length $N_l/2$ are available to perform four independent ACs.

The AC with sliding and averaging windows of $W_l = N_l/4$ samples can be calculated according to

$$p_{j,q}[n] = \frac{1}{W_l}\sum_{k=0}^{W_l-1} r_{j,q}[n + k + W_l] r_{j,q}^*[n + k], \tag{7}$$

where $j \in \{u, d\}$ and $q \in \{1, 2\}$. Neglecting the noise terms for simplicity, the outputs of the autocorrelators at the maximum of their absolute values can be expressed as

$$p_{j,1}[n_{j,1,\max}] = |h|^2 (-1)^b e^{j\pi\epsilon N_l}, \tag{8}$$

$$p_{j,2}[n_{j,2,\max}] = |h|^2 (-1)^{b+1} e^{j\pi\epsilon N_l}, \tag{9}$$

where $b = \mathrm{mod}(W_l, 2)$, with $\mathrm{mod}(.)$ being the modulo operator. The corresponding phases

$$\phi_{j,1} = \angle p_{j,1}[n_{j,1,\max}] = \pi\epsilon N_l + b\pi, \tag{10}$$

$$\phi_{j,2} = \angle p_{j,2}[n_{j,2,\max}] = \pi\epsilon N_l + (1 - b)\pi, \tag{11}$$

can be used to estimate the individual FFOs as

$$\hat{\epsilon}_{j,1} = \frac{1}{N_l}\left[\frac{1}{\pi}\phi_{j,1} - b\right], \quad \hat{\epsilon}_{j,1} \in \frac{1}{N_l}\left[-(1 + b), (1 - b)\right], \tag{12}$$

$$\hat{\epsilon}_{j,2} = \frac{1}{N_l}\left[\frac{1}{\pi}\phi_{j,2} - (1 - b)\right], \quad \hat{\epsilon}_{j,2} \in \frac{1}{N_l}\left[(b - 2), b\right]. \tag{13}$$

In order to achieve an estimation with high accuracy, the information provided by all individual phases should be used. However, since $\hat{\epsilon}_{j,1}$ and $\hat{\epsilon}_{j,2}$ lie in different definition domains, a simple average of the phases would reduce the effective estimation range of the FFO to $[-1/N_l, 0]$. This effect can be avoided with the modification of the phases obtained from the second demultiplexed signal as $\phi'_{j,2} = \phi_{j,2} - \pi$. The FFO can thus be obtained as

$$\hat{\epsilon}_{F,l} = \frac{\phi_{u,1} + \phi_{d,1} + \phi'_{u,2} + \phi'_{d,2}}{4\pi N_l}. \tag{14}$$

Demultiplexing the dual-chirp enables the computation of two ACs, where the sliding and averaging windows consist of $W_c = N_c/4$ samples. The maxima of the ACs are given by

$$p_{c,1}[n_{1,\max}] = |h|^2 \alpha^2 (-1)^b e^{j\pi\epsilon N_c}, \tag{15}$$

$$p_{c,2}[n_{2,\max}] = |h|^2 \alpha^2 (-1)^{b+1} e^{j\pi\epsilon N_c}. \tag{16}$$

The same reasoning as in the previous case leads to an estimation of the FFO as

$$\hat{\epsilon}_{F,c} = \frac{\phi_1 + \phi_2'}{2\pi N_c}, \tag{17}$$

where $\phi_1$ corresponds to the phase of $p_{c,1}[n_{1,\max}]$ and $\phi_2'$ refers to the phase of $p_{c,2}[n_{2,\max}]$ after phase compensation.

### B. Integer Frequency Offset Estimation

After FFO compensation, the IFO can be found using the information provided by the outputs of the MFs.

The impulse response $g_{i,j}[n]$ of the MF matched to the signal $x_j[n]$ can be formulated as

$$g_{i,j}[n] = \beta_i x_j^*[N_i - 1 - n], \tag{18}$$

where $i \in \{l, c\}$, $j \in \{u, d\}$, and

$$\beta_i = \begin{cases} 1, & i = l, \\ 1/\alpha, & i = c. \end{cases} \tag{19}$$

The convolution between the signal $r_{\epsilon_f}^{[s]}[n]$ and the matched filter $g_{i,j}[n]$ is defined as

$$m_{i,j}[n] = \frac{1}{N_i} \sum_{k=-\infty}^{\infty} r_{\epsilon_f}^{[s]}[k] g_{i,j}[n-k], \tag{20}$$

where $r_{\epsilon_f}^{[s]}[n]$ is the reference signal after proper compensation of the FFO $\hat{\epsilon}_F$, and it can be described as

$$r_{\epsilon_f}^{[s]}[n] = r[n + s_{i,j} N_i] e^{-j2\pi\hat{\epsilon}_{F,i}(n+s_{i,j}N_i)}, \tag{21}$$

where

$$s_{i,j} = \begin{cases} 0, & (i = l \wedge j = u) \vee i = c, \\ 1, & (i = l \wedge j = d), \end{cases} \tag{22}$$

with $\wedge$ and $\vee$ as the logical operators "and" and "or", respectively.

After some algebraic manipulations, the magnitudes of the outputs of the MFs can be written as

$$|m_{i,u}[n_i']| = |h|(1 - |n_i'|) \left| \frac{\mathrm{sinc}\left(N_i(n_i' + \epsilon_{I,i})(1 - |n_i'|)\right)}{\mathrm{sinc}(n_i' + \epsilon_{I,i})} \right|, \tag{23}$$

$$|m_{i,d}[n_i'']| = |h|(1 - |n_i''|) \left| \frac{\mathrm{sinc}\left(N_i(n_i'' - \epsilon_{I,i})(1 - |n_i''|)\right)}{\mathrm{sinc}(n_i'' - \epsilon_{I,i})} \right|, \tag{24}$$

where the new discrete time indexes, $n_i'$ and $n_i''$, are given by

$$n_i' = \frac{n - N_i + 1}{N_i}, \quad \text{and} \quad n_i'' = \frac{n - \gamma_i N_i + 1}{N_i}, \tag{25}$$

with

$$\gamma_i = \begin{cases} 2, & i = l, \\ 1, & i = c, \end{cases} \tag{26}$$

and the frequency offset $\epsilon_{I,i}$ represents the IFO (plus some residual FFO, which will be neglected for this analysis), normalized by the sampling frequency.

The arguments of the outputs of the MFs at their maximum correspond to

$$\hat{n}_{i,u} = \arg\max_n |m_{i,u}[n_i']| = -N_i\epsilon_{I,i} + N_i - 1, \tag{27}$$

$$\hat{n}_{i,d} = \arg\max_n |m_{i,d}[n_i'']| = N_i\epsilon_{I,i} + \gamma_i N_i - 1, \tag{28}$$

which can, ideally, be combined to estimate the IFO. However, the presence of uncompensated FFOs of magnitude close to half of the IFO spacing ($f_s/N_i$), distorts the ideal behavior of the MF. Its output is not characterized by a clearly defined maximum, but by two values of similar amplitude. In this case, choosing the index of the highest value can lead to ambiguities in the estimation of the IFO. An accurate estimation should consider the position of the first value exceeding a certain threshold at the output of each MF. The IFO can thus be estimated with an accuracy of $f_s/(2N_i)$, avoiding potential ambiguities due to uncompensated FFOs. The new indexes, after ambiguity resolution, provide an estimation of the IFO as

$$\hat{\epsilon}_{I,i} = \frac{\hat{n}_{i,d} - \hat{n}_{i,u} - (\gamma_i - 1)N_i}{2N_i}. \tag{29}$$

Hence, the overall FO estimated with each sequence consists of two terms, as $\hat{\epsilon}_i = \hat{\epsilon}_{I,i} + \hat{\epsilon}_{F,i}$.

### C. DFT-Based Frequency Offset Estimation

The approach presented in [3] exploits the properties of the DFT for the estimation of the FO. After multiplying the time-synchronized received signal with the complex conjugate of each reference chirp, $\beta_i x_j^*[n]$, two DFTs with an upsampling factor are performed. The maxima obtained at the outputs of the DFTs are used for the estimation of the FO, similarly to the approach described in time domain, making use of the time-frequency duality of the DFT.

## IV. CRLBs For Frequency Offset Estimation

The likelihood function (LF) of the received vector $\mathbf{r}$, with $N_i$ independent observations in complex AWGN, can be expressed as [5]

$$\mathrm{p}(\mathbf{r};\boldsymbol{\xi}) = \frac{1}{\pi^N \det(\mathbf{C}_{r(\xi)})} e^{\left[-(\mathbf{r}-\mathbf{s}(\xi))^H \mathbf{C}_{r(\xi)})^{-1}(\mathbf{r}-\mathbf{s}(\xi))\right]}, \tag{30}$$

where $\mathbf{C}_{r(\xi)} = \sigma^2\mathbf{I}$ is the covariance matrix of the received signal $r[n]$, and the components of $\mathbf{s}(\xi)$ are given by

$$s[n] = h_0 x[n] e^{j2\pi\epsilon n} e^{j\phi_h}, \qquad n = 0, 1, ..., N_i - 1, \tag{31}$$

where $h_0$ and $\phi_h$ are, respectively, the magnitude and the argument of the complex channel gain $h$, and $x[n]$ corresponds to the transmitted reference sequence of length $N_i$ samples, which will be properly specified in the next subsections.

The second derivative of the log-likelihood function (LLF) of $\mathbf{r}$ with respect to the parameters $p$ and $q$, provides the $\{p, q\}$-element of the Fisher information matrix, which can be calculated as [5]

$$[\mathbf{I}]_{p,q} = \frac{2}{\sigma^2} \mathrm{Re}\left[\sum_{n=0}^{N_l-1} \frac{\partial s^*[n]}{\partial \xi_p} \frac{\partial s[n]}{\partial \xi_q}\right], \tag{32}$$

where $p, q \in \{1, 2, 3\}$, refer to the components of the vector of unknown real parameters $\boldsymbol{\xi} = [h_0 \ \epsilon \ \phi_h]^T$.

### A. CRLB for Linear Up-Down-Chirp

For the derivation of the CRLB of the FO attainable with a linear up-chirp, the signal $x[n]$ equals $x_u[n]$, for $n = 0, 1, ..., N_l - 1$. The Fisher information matrix is given by

$$\mathbf{I} = \frac{2N_l}{\sigma^2}\begin{bmatrix}1 & 0 & 0\\ 0 & \frac{2\pi^2 h_0^2(N_l-1)(2N_l-1)}{3} & \pi h_0^2(N_l-1)\\ 0 & \pi h_0^2(N_l-1) & h_0^2\end{bmatrix}. \quad (33)$$

With $\text{SNR} = h_0^2/\sigma^2$, and after some algebraic manipulations, the minimum variance associated to the up-chirp can be expressed as

$$Var(\hat{\epsilon}_{l,u}) \geq [\mathbf{I}^{-1}]_{22} = \frac{3}{2\pi^2 N_l(N_l^2-1)\text{SNR}}, \quad (34)$$

which is the same as the variance attainable with the down-chirp. Therefore, due to the additivity property of the information for independent observations [6], the variance obtained with the training sequence consisting of one up-chirp followed by one down-chirp results in

$$Var(\hat{\epsilon}_l) = \frac{1}{2}Var(\hat{\epsilon}_{l,u}). \quad (35)$$

### B. CRLB for Dual-Chirp

In this case, $x[n] = x_c[n]$, for $n = 0, 1, ..., N_c - 1$. The same procedure as in the preceding case yields the new Fisher information matrix

$$\mathbf{I} = \frac{2}{\sigma^2}\begin{bmatrix}\alpha^2 U & 0 & 0\\ 0 & 4\pi^2 h_0^2\alpha^2 W & 2\pi h_0^2\alpha^2 V\\ 0 & 2\pi h_0^2\alpha^2 V & \alpha^2 h_0^2 U\end{bmatrix}. \quad (36)$$

After applying the following approximations,

$$U = \sum_{n=0}^{N_c-1}\cos^2\left(\pi\left(\frac{n^2}{N_c} - n\right)\right) \approx \frac{N_c}{2}, \quad (37)$$

$$V = \sum_{n=0}^{N_c-1} n\cos^2\left(\pi\left(\frac{n^2}{N_c} - n\right)\right) \approx \frac{N_c(N_c-1)}{4}, \quad (38)$$

$$W = \sum_{n=0}^{N_c-1} n^2\cos^2\left(\pi\left(\frac{n^2}{N_c} - n\right)\right) \approx \frac{N_c(N_c-1)(2N_c-1)}{12}, \quad (39)$$

the lower bound for the variance of the FO obtainable with a dual-chirp of length $N_c$ samples can be written as

$$Var(\hat{\epsilon}_c) \geq [\mathbf{I}^{-1}]_{22} \approx \frac{3}{4\pi^2\alpha^2 N_c(N_c^2-1)\text{SNR}}. \quad (40)$$

With $N_c = \rho N_l$, the ratio between the CRLBs achievable with both training sequences can be expressed as

$$R = \frac{Var(\hat{\epsilon}_l)}{Var(\hat{\epsilon}_c)} = \frac{\alpha^2\rho(\rho^2 N_l^2 - 1)}{(N_l^2-1)}. \quad (41)$$

## V. SIMULATION RESULTS

This section presents the performance evaluation of the proposed synchronization approach described in Section III. For this purpose, simulations have been conducted and compared to the theoretical references provided by the derived CRLBs. The sweeping frequency $B$ of the reference signal is set to 180 kHz. The system is assumed to work at the Nyquist rate. For a fair comparison, both reference signals are transmitted with the same power and have the same length, with $N_l = 64$ and $N_c = 128$ samples. $N_l$ and $N_c$ are chosen to ensure integer compression factors after downsampling the sequences. During the simulations, a uniformly distributed FO in the range $[-18, 18]$ kHz has been generated, corresponding to a transmission in the 900 MHz band and an accuracy of the local oscillators of 20 ppm.

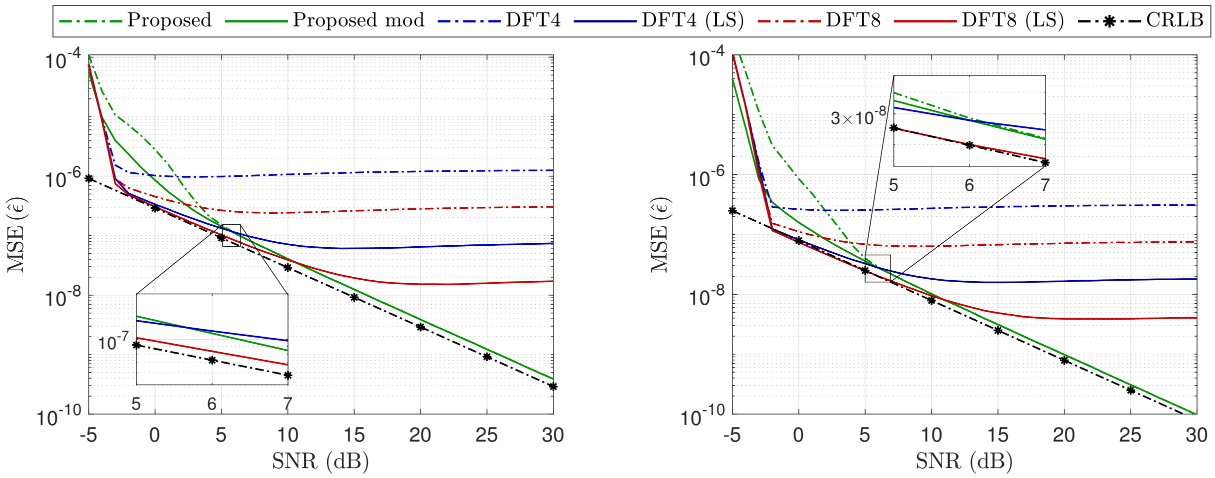
In order to compare the performance of the proposed approach with the state-of-the-art algorithm [3], the latter has been implemented and simulated with upsampling factors 4 and 8. Additionally, least-squares (LS) based second-order-polynomial interpolation with an upsampling factor 4 has been incorporated to increase the accuracy of the estimation. The coefficients of the second-order-polynomial are calculated according to the least-squares principle, based on three samples selected at the output of the DFT, namely the highest one, the preceding, and the following.

The metric selected for the performance evaluation is the mean squared error (MSE) of the estimation, defined as $\text{MSE} = \mathbf{E}\left[(\epsilon - \hat{\epsilon})^2\right]$, where $\epsilon$ and $\hat{\epsilon}$ are normalized by the sampling frequency.

The simulation results obtained with the up-down-chirp and with the dual-chirp are depicted in Fig. 1(a) and 1(b), respectively. Even though they show similar behavior, there is a clear difference in the magnitude of the MSE. This effect is in accordance with the theoretical ratio between variances given in (41), which in this case has a value of 5.65 dB. It can be shown that, for $\rho = 2$ and long sequences, $\alpha \approx 1/\sqrt{2}$, and the ratio $R$ tends to 6 dB. The main reason for the superiority of the dual-chirp is the higher compression factor of its underlying linear reference sequences.

The estimation obtained with the proposed approach achieves an MSE close to the CRLB for SNR values above 5 dB. In the low SNR regime, the MSE is significantly improved through the phase compensation described in III-A (Proposed mod). The performance achievable with the DFT-approach highly depends on the upsampling factor and the LS interpolation. Especially the latter operation can substantially improve the accuracy of the estimation, and provide, in a limited SNR range, an MSE lower than the proposed approach. Nevertheless, the MSE obtained with the DFT-approach shows an error floor, which can only be reduced through the use of higher upsampling factors and interpolation. Consequently, a performance improvement leads to an increase in latency and computational complexity.

Table I lists the computational complexity of each approach in terms of number of complex multiplications (CMs). The proposed approach considers the CMs associated to the ACs and MFs. In case of the up-down-chirp, four ACs of length $N_l/4$ are calculated, whilst with the dual-chirp, two ACs of length $N_c/4$ are computed. For the sake of efficiency, matched filtering has been implemented in frequency domain.




(a) Linear up-down-chirp

(b) Dual-chirp

Fig. 1: MSE of the frequency offset estimation obtained with (a) a linear up-down-chirp and (b) a dual-chirp.

Appropriate zero padding has been considered to achieve equivalence between linear and circular convolution, and enable a DFT operation based on Radix-2 fast Fourier transform (FFT). These assumptions lead to the new extended lengths, $N_{l,ext} = 128$, and $N_{c,ext} = 256$ samples.

The DFT-based approach includes the initial multiplication with the complex conjugate of each linear reference signal, the complexity due to the DFT operation (assuming Radix-2 FFT implementation), and the complexity required for the LS interpolation ($C_{LS}$). The latter includes the operations needed to find the coefficients of the second-order polynomial, evaluate the polynomial at the newly defined positions, and find the maximum.

Considering the simulation parameters, the DFT-based approach with upsampling factor $8$ requires approximately $40\%$ more CMs than the proposed approach. A further improvement of the accuracy achieved with the state-of-the-art algorithm, through the use of higher upsampling factors, would increase significantly the number of CMs, making the difference in terms of complexity between both approaches more evident.

The compression factor of the reference signal, is responsible, not only for the accuracy of the estimation, but also for the complexity. Therefore, a higher compression factor selected for the dual-chirp, compared to the up-down-chirp, implies a higher number of CMs, independently of the adopted algorithm.

TABLE I: Complexity of the presented approaches.

| Approach | Complexity (CMs) |
|---|---|
| Proposed [1] | $0.5\gamma_i N_i + 2\,(1.5\log_2(N_{i,ext}) + 1)$ |
| DFT | $2N_i + f_{up}N_i\log_2(f_{up}N_i) + 2\,C_{LS}$ |

[1] $\gamma_i$ as defined in (26)

## VI. Conclusions

In this work, a chirp-based frequency synchronization approach, suitable for flat fading channels under high FOs, has been proposed and examined in detail. Its performance has been evaluated by means of simulations, compared to the state-of-the-art approach, and validated with theoretical limits provided by CRLBs. Aiming at low complexity, the synchronization process can be undertaken at a low sampling rate, avoiding upsampling and interpolation. Nevertheless, a high accuracy is achieved, and the approach exhibits near-optimum performance for SNR values above 5 dB.

Two training sequences have been considered for the evaluation. Under the assumption of identical length and transmit power, it is shown that the dual-chirp achieves a lower MSE than the linear up-down-chirp, at the cost of higher computational complexity.

The ratio between the derived CRLBs and the complexity analysis can be used as tools for the design and selection of the most suitable reference sequence that allows to meet specific system requirements.

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