Performance Assessment of Orthogonal Chirp Division Multiplexing in MIMO Space Time Coding

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***Abstract*—Recent results have shown that the recently proposed Orthogonal Chirp Division Multiplexing (OCDM) system outperforms OFDM under frequency-selective channels in single-input single-output case. In this work, we investigate the viability of OCDM in a multiple-input multiple-output case with different space time coding techniques. In particular, we consider: Alamouti, cyclic delay diversity (CDD) and Alamouti with CDD (ACDD) techniques. The contribution of this paper is twofold. Firstly, we provide a comprehensive model for ACDD, which for the best of the authors' knowledge is not available in the literature. Secondly, we evaluate this model in a vehicular communication scenario based on ITS-G5 standard, and we show that OCDM outperforms OFDM in terms of frame error rate performance.**

***Index Terms*—Space-time coding, Alamouti, cyclic delay diversity, OCDM.**

I. Introduction

It is well known that the waveform design plays a significant role on the performance of a wireless communication system. For instance, the authors in [1] show that orthogonal frequency division multiplexing (OFDM) is suboptimal under frequency-selective channels (FSCs) when channel state information (CSI) is available at the receiver. On the other hand, the recently proposed orthogonal chirp division multiplexing (OCDM) scheme [2] theoretically provides optimal performance assuming the employment of an iterative receiver, which is capable of achieving the performance of perfect-feedback equalizer (PFE). In [3], a low-complexity iterative receiver based on the mininmum mean squared error with parallel iterference cancellation (MMSE-PIC) has been proposed for OCDM, and has been shown to provide very close performance to the PFE.

In addition to the waveform design, multiple-input multiple-output (MIMO) technology also improve the system's robustness considerably by means of space-time coding (STC) [4]. The receiver diversity is relatively straightforward to achieve by the maximum-ratio combining (MRC) technique, where the signal-to-noise ratio (SNR) is increased after proper combination of signals from different receiver antennas [5]. Conversely, the transmit diversity is more challenging. In the 90's, several transmit diversity schemes emerged. For instance, techniques based on delay diversity were introduced in [6], [7], that can be termed cyclic delay diversity (CDD) if we consider cyclic shift as shown in [8]. Later, a simple technique was proposed by Alamouti for two transmit antennas [9], which is straightforwardly applicable for nowadays systems under FSCs by frequency-domain processing [10]. In order to apply it for a higher amount of transmit antennas, the Alamouti scheme was combined with CDD in [11], which can be termed as Alamouti with cyclic delay diversity (ACDD). In short, the Alamouti's scheme in frequency domain can be arranged as the input of a CDD block in a very simple manner.

The contribution of this paper is twofold. Firstly, we provide a detailed model in frequency domain for the ACDD scheme based on Alamouti and CDD models. Secondly, we utilize the STC models to extend the evaluation done in [3] by assessing the performance of OCDM in a multiple antennas environment, and comparing it to the state-of-the art OFDM. Since our evaluation is particularly interesting for ultra reliable low-latency communications (URLLC), we apply the STC schemes to vehicular communication based on the ITS-G5 standard [12], [13], which is originally based on OFDM, thereby demonstrating the feasibility of OCDM for such system. As expected, the results show that ACDD with 8 transmit antennas improves significantly the performance of Alamouti, leading to an SNR gain of approximately 4 and 2 dB with frame error rate (FER) of 10^{-5}, when one and two received antenna are employed, respectively. Therefore it is clear the potential of ACDD for URLLC applications. Moreover, we show that as the amount of transmit antennas increase, the performance gap between OCDM and OFDM enlarges, demonstrating the potential of OCDM for STC MIMO. This behavior occurs due to extra frequency selectivity introduced by the CDD component of ACDD.

Notation: the operator $\mathbb{E}(\cdot)$ denotes the expected value of its argument. The special matrices $\mathbf{I}_N$ and $\mathbf{F}_N$ with size $N \times N$ denote the identity and normalized Fourier matrix, respectively. The operators $(\cdot)^\dagger$ and $(\cdot)^\mathrm{H}$ denote conjugate and hermitian, respectively. $\mathbf{a} \circledast \mathbf{b}$ is the circular convolution between $\mathbf{a}$ and $\mathbf{b}$ [14, eq. (9.6.6)], where the smaller vector between $\mathbf{a}$ and $\mathbf{b}$ is completed with zeros in case they have different lengths. The operation $\mathrm{diag}(\mathbf{a})$ returns a diagonal matrix whose elements are obtained from the vector $\mathbf{a}$. The operator $(\mathbf{a})_{n\in\mathcal{A}}$ returns the elements of $\mathbf{a}$ whose indexes belong to the set $\mathcal{A}$.

The rest of this paper is organized as follows. The Alamouti, CDD and ACDD transmit diversity system models are presented in Section II. In Section III, the models are extended to encompass multiple-antennas at receiver. After that, we briefly comment about data estimation in Section IV. Subsequently, the numerical results are presented in Section V in order to analyze the feasibility of OCDM in a MIMO system. Finally, Section VI concludes the paper.

II. Transmit Diversity Schemes

In this section, we present the models for three diversity schemes considered in this paper. In particular, we provide a comprehensive model for the ACDD STC technique, which has been introduced in [11], but for the best of the authors' knowledge, it has not yet been formally defined in frequency-domain nor presented together with receive diversity. In addition, the relevant characteristic of these STC schemes is that the MIMO channel models become equivalent to a single-input single-output (SISO) channel model, thereby facilitating the signal processing at the receiver and the comprehension of the overall system. This aspect is easily shown by our model in Section III.

In addition, we highlight that we provide a general model that is applicable regardless of the waveform or transmitter matrix.

A. Alamouti STC

We consider the Alamouti's STC in frequency domain with two transmit antennas and one receive antenna. Let us first define two streams of data $\mathbf{d}_0, \mathbf{d}_1 \in \mathcal{S}^{K_{\text{on}}}$, with a covariance matrix given by $\mathbb{E}(\mathbf{d}_0\mathbf{d}_0^{\text{H}}) = \mathbb{E}(\mathbf{d}_1\mathbf{d}_1^{\text{H}}) = E_{\text{s}}/2\,\mathbf{I}_{K_{\text{on}}}$, i.e., each data stream consumes half of transmit energy E_{s} due to simultaneous transmissions. $\mathcal{S}$ represents a quadrature amplitude modulation (QAM) constellation set with cardinality $|\mathcal{S}| = J$, and K_{on} is the number of allocated subcarriers.

In general, the transmitted signals in frequency domain for each data stream are given by

$$\mathbf{X}_0 = \mathbf{A}\mathbf{d}_0 \qquad \text{and} \qquad \mathbf{X}_1 = \mathbf{A}\mathbf{d}_1, \tag{1}$$

being $\mathbf{A} \in \mathbb{C}^{K_{\text{on}} \times K_{\text{on}}}$ the modulation matrix in frequency domain with $\mathbf{A}^{\text{H}}\mathbf{A} = \mathbf{I}$. Thus, Alamouti's STC [9] is directly applied to the elements of $\mathbf{X}_0$ and $\mathbf{X}_1$ as shown in Table I [10], where "Time 0" and "Time 1" denote the period of first and second block transmission, respectively. The diagram for block-based Alamouti's STC is depicted in the diagram a) of Fig. 1. The $\mathbf{F}_N^{\text{H}}$ blocks perform the inverse fast Fourier transform (FFT) of size N, where $N - K_{\text{on}}$ subcarriers at the edges of the spectrum are set to zero. The CP block adds the cyclic prefix of length N_{cp}, which is assumed to be greater than the channel length. The delay block retains its input for a period of $N{+}N_{\text{cp}}$ samples in order to delay the branch to the next time slot.

TABLE I
Alamouti's STC

	Antenna 0	Antenna 1
Time 0	$\mathbf{X}_0$	$\mathbf{X}_1$
Time 1	$-\mathbf{X}_1^{\dagger}$	$\mathbf{X}_0^{\dagger}$

Assuming a synchronized system for simplicity and an invariant channel over the transmission of 2 blocks, the received signal with one receive antenna is then given by [10]

$$\mathbf{Y}_{\text{Al}} = \begin{bmatrix} \mathbf{Y}^{\text{t}_0} \\ (\mathbf{Y}^{\text{t}_1})^{\dagger} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{\Lambda}_0 & \mathbf{\Lambda}_1 \\ \mathbf{\Lambda}_1^{\text{H}} & -\mathbf{\Lambda}_0^{\text{H}} \end{bmatrix}}_{\mathbf{\Lambda}_{\text{Al}}} \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{X}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{W}^{\text{t}_0} \\ (\mathbf{W}^{\text{t}_1})^{\dagger} \end{bmatrix}, \tag{2}$$

where $\mathbf{Y}^{\text{t}_0}, \mathbf{Y}^{\text{t}_1} \in \mathbb{C}^{K_{\text{on}}}$ are the received signals in frequency-domain at transmission times 0 and 1, respectively. $\mathbf{\Lambda}_0, \mathbf{\Lambda}_1 \in \mathbb{C}^{K_{\text{on}} \times K_{\text{on}}}$ are diagonal matrices, and whose elements are the frequency-domain responses of channels related to antennas 0 and 1, respectively[1]. $\mathbf{\Lambda}_{\text{Al}} \in \mathbb{C}^{2K_{\text{on}} \times 2K_{\text{on}}}$ is the equivalent channel matrix which is orthogonal [10]. Finally, $\mathbf{W}^{\text{t}_0}, \mathbf{W}^{\text{t}_1} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{K_{\text{on}}})$ are vectors with independent and identically distributed additive white Gaussian noise samples with zero mean and variance σ^2.

Due to orthogonality of $\mathbf{\Lambda}_{\text{Al}}$, it is shown in [10] that multiplying both sides of (2) by $\mathbf{\Lambda}_{\text{Al}}^{\text{H}}$ decouples the data vectors $\mathbf{X}_0$ and $\mathbf{X}_1$. Then, assuming perfect channel knowledge at the receiver, the model becomes

$$\begin{aligned} \tilde{\mathbf{Y}}_{\text{Al}} &= \frac{\mathbf{\Lambda}_{\text{Al}}^{\text{H}}}{\sqrt{\mathbf{\Lambda}_{\text{Al}}^{\text{H}}\mathbf{\Lambda}_{\text{Al}}}} \mathbf{Y}_{\text{Al}} \\ &= \begin{bmatrix} \tilde{\mathbf{\Lambda}}_{\text{Al}} & \\ & \tilde{\mathbf{\Lambda}}_{\text{Al}} \end{bmatrix} \begin{bmatrix} \mathbf{A} & \\ & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \end{bmatrix} + \tilde{\mathbf{W}}, \end{aligned} \tag{3}$$

where $\tilde{\mathbf{\Lambda}}_{\text{Al}} = \sqrt{|\mathbf{\Lambda}_0|^2 + |\mathbf{\Lambda}_1|^2}$ is a diagonal matrix and $\tilde{\mathbf{W}} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{2K_{\text{on}}})$ is the noise related signal. By equations (2) and (3), one easily shows that covariance matrix of $\tilde{\mathbf{W}}$ is $\sigma^2(\mathbf{\Lambda}_{\text{Al}}^{\text{H}}/\sqrt{\mathbf{\Lambda}_{\text{Al}}^{\text{H}}\mathbf{\Lambda}_{\text{Al}}})(\mathbf{\Lambda}_{\text{Al}}/\sqrt{\mathbf{\Lambda}_{\text{Al}}^{\text{H}}\mathbf{\Lambda}_{\text{Al}}}) = \sigma^2\mathbf{I}_{2K_{\text{on}}}$. Clearly, the model of (3) shows that the $2{\times}1$ multiple-input single-output (MISO) system with Alamouti STC becomes a SISO whose equivalent channel is the average of channels 0 and 1.

B. Cyclic Delay Diversity (CDD)

For the CDD scheme, it is not necessary to use more than one time slot for transmission, then we consider only one data vector $\mathbf{d} \in \mathcal{S}^{K_{\text{on}}}$ with covariance matrix given by $\mathbb{E}(\mathbf{d}\mathbf{d}^{\text{H}}) = E_{\text{s}}/N_{\text{t}}\,\mathbf{I}_{K_{\text{on}}}$, being N_{t} the amount of transmit antennas. Then, the transmit signal in frequency domain is

$$\mathbf{X} = \mathbf{A}\mathbf{d}. \tag{4}$$

Furthermore, notice that CDD allows more than 2 transmit antennas. The CDD diagram described in [8] is depicted in part b) of Fig. 1. In this case, it is more convenient to first

[1] Notice that $\mathbf{\Lambda}_0$ and $\mathbf{\Lambda}_1$ consider only the responses of the allocated subcarriers.

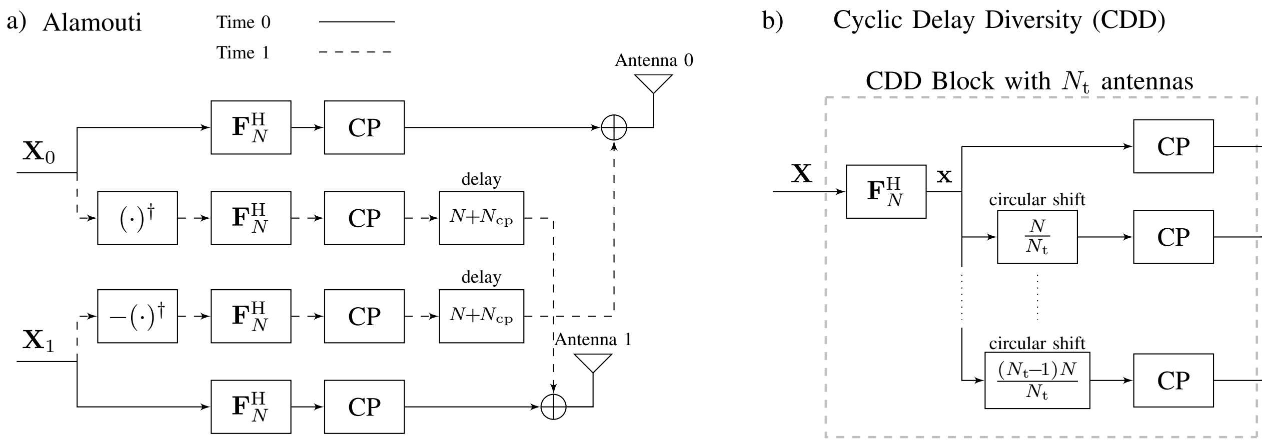

Fig. 1. Block diagrams of a) Alamouti, and b) Cyclic Delay Diversity STC schemes for OFDM.

express the received signal in time domain, which for one receive antenna is given by

$$\begin{aligned}
\mathbf{y}_{\mathrm{CDD}} &\overset{(a)}{=} \sum_{n_{\mathrm{t}}=0}^{N_{\mathrm{t}}-1} \left(\mathbf{x} \circledast \boldsymbol{\delta}_{(n_{\mathrm{t}}N/N_{\mathrm{t}})}\right) \circledast \mathbf{h}_{n_{\mathrm{t}}} + \mathbf{w}, \\
&\overset{(b)}{=} \mathbf{x} \circledast \left(\sum_{n_{\mathrm{t}}=0}^{N_{\mathrm{t}}-1} \boldsymbol{\delta}_{(n_{\mathrm{t}}N/N_{\mathrm{t}})} \circledast \mathbf{h}_{n_{\mathrm{t}}}\right) + \mathbf{w}. \qquad (5) \\
&\overset{(c)}{=} \mathbf{x} \circledast \tilde{\mathbf{h}} + \mathbf{w}.
\end{aligned}$$

where $\mathbf{x} \in \mathbb{C}^N$ is the output of the inverse FFT block in part b) of Fig. 1. Again, the inverse FFT block sets the subcarriers in the edges of the spectrum to zeros for proper subcarrier allocation. The circular shift block in part b) of Fig. 1 performs the operation $\mathbf{x} \circledast \boldsymbol{\delta}_{(n_{\mathrm{t}}N/N_{\mathrm{t}})}$ defined in part (a) of equation (5), for the respective transmit antenna. $\mathbf{h}_{n_{\mathrm{t}}} \in \mathbb{C}^{L_{\mathrm{ch}}}$ is the impulse response of the channel between the n_{t}th transmit antenna and the receiver, with length L_{ch} that is assumed to be smaller than N_{cp} and equal for all n_{t}. And $\boldsymbol{\delta}_{(i)}$ is a vector with N elements, whose ith position is equal to 1 all the other positions are zero, which are defined as

$$\boldsymbol{\delta}_{(i)}[n] = \begin{cases} 1, & n = i \\ 0, & n \neq i \end{cases} \qquad n = 0, 1, 2, \cdots, N-1. \qquad (6)$$

Thus, the operation $\mathbf{x} \circledast \boldsymbol{\delta}_{(i)}$ represents the circular shift of $\mathbf{x}$ by i positions. By swapping the convolution order, the MISO becomes equivalent to a SISO system as shown in part (b) of equation (5), where the equivalent channel $\tilde{\mathbf{h}} \in \mathbb{C}^N$ is

$$\tilde{\mathbf{h}} = \sum_{n_{\mathrm{t}}=0}^{N_{\mathrm{t}}-1} \boldsymbol{\delta}_{(n_{\mathrm{t}}N/N_{\mathrm{t}})} \circledast \mathbf{h}_{n_{\mathrm{t}}}. \qquad (7)$$

In frequency domain, we finally have

$$\tilde{\mathbf{Y}}_{\mathrm{CDD}} = \tilde{\boldsymbol{\Lambda}}_{\mathrm{CDD}} \mathbf{A} \mathbf{d} + \tilde{\mathbf{W}}_{\mathrm{CDD}}, \qquad (8)$$

where $\tilde{\boldsymbol{\Lambda}}_{\mathrm{CDD}} = \mathrm{diag}(\tilde{\mathbf{h}}_{\mathrm{F}}) \in \mathbb{C}^{K_{\mathrm{on}} \times K_{\mathrm{on}}}$ is a diagonal matrix whose elements represent the channel response in frequency domain for the set of allocated subcarriers $\mathcal{K}_{\mathrm{on}}$, i.e., $\tilde{\mathbf{h}}_{\mathrm{F}} = (\mathbf{F}_N \tilde{\mathbf{h}})_{k \in \mathcal{K}_{\mathrm{on}}}$, and $\tilde{\mathbf{W}}_{\mathrm{CDD}} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{K_{\mathrm{on}}})$ is the noise related term.

C. Alamouti with CDD (ACDD)

In [11], Alamouti and CDD techniques are combined in order to increase the transmit diversity. In this paper, we refer to this scheme as ACDD. ACDD is illustrated in Fig. 2. Basically, we use the data signals of table I as input for the CDD Blocks. In contrast to only two antennas in Alamaouti's scheme, the new arrangement employs two *clusters* of antennas, where each cluster is organized in a CDD fashion. Thus, we can straightforwardly define the model for this arrangement in

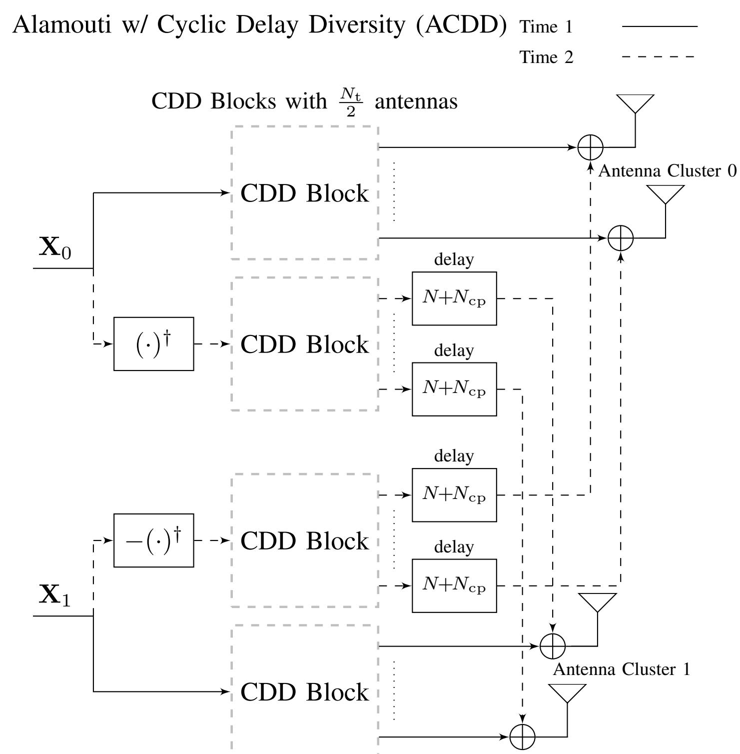

Fig. 2. Block diagram of the proposed scheme: Alamouti CDD (ACDD).

analogy to equation (3) by

$$\tilde{\mathbf{Y}}_{\mathrm{ACDD}} = \begin{bmatrix} \tilde{\mathbf{\Lambda}}_{\mathrm{ACDD}} & \\ & \tilde{\mathbf{\Lambda}}_{\mathrm{ACDD}} \end{bmatrix} \begin{bmatrix} \mathbf{A} & \\ & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \end{bmatrix} + \tilde{\mathbf{W}}_{\mathrm{ACDD}}, \tag{9}$$

where $\tilde{\mathbf{\Lambda}}_{\mathrm{ACDD}} = \sqrt{\mathrm{diag}(|\tilde{\mathbf{h}}_{0_\mathrm{F}}|^2 + |\tilde{\mathbf{h}}_{1_\mathrm{F}}|^2)} \in \mathbb{C}^{K_{\mathrm{on}} \times K_{\mathrm{on}}}$ is obtained by combining models of equations (3) and (8) and represents a diagonal matrix whose elements are the channel response in frequency domain for the allocated subcarriers, i.e., $\tilde{\mathbf{h}}_{0_\mathrm{F}} = (\mathbf{F}_N \tilde{\mathbf{h}}_0)_{k \in \mathcal{K}_{\mathrm{on}}}$ and $\tilde{\mathbf{h}}_{1_\mathrm{F}} = (\mathbf{F}_N \tilde{\mathbf{h}}_1)_{k \in \mathcal{K}_{\mathrm{on}}}$. In addition, $\tilde{\mathbf{h}}_0$ and $\tilde{\mathbf{h}}_1$ are defined analogously to (7) as

$$\tilde{\mathbf{h}}_0 = \sum_{n_\mathrm{t}=0}^{N_\mathrm{t}/2-1} \boldsymbol{\delta}_{(n_\mathrm{t} 2N/N_\mathrm{t})} \circledast \mathbf{h}_{n_\mathrm{t}} \tag{10}$$

and

$$\tilde{\mathbf{h}}_1 = \sum_{n_\mathrm{t}=0}^{N_\mathrm{t}/2-1} \boldsymbol{\delta}_{(n_\mathrm{t} 2N/N_\mathrm{t})} \circledast \mathbf{h}_{(n_\mathrm{t}+N_\mathrm{t}/2)}. \tag{11}$$

In this case, $\tilde{\mathbf{h}}_0$ and $\tilde{\mathbf{h}}_1$ are the equivalent channel impulse responses of clusters 0 and 1, respectively, which are obtained by the CDD scheme with $N_\mathrm{t}/2$ antennas. The noise related term in (9) is obtained analogously to (3) and is $\tilde{\mathbf{W}}_{\mathrm{ACDD}} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{2K_{\mathrm{on}}})$. Finally, it is worth noting that ACDD can be seen as a generalization of Alamouti, where Alamouti is obtained for $N_\mathrm{t} = 2$.

Finally, we notice that the channel estimation of ACDD is more challenging than in Alamouti's case, because the channel becomes more selective due to the artificial extension of its power delay profile, which is directly observed by equations (10) and (11). Designing a specific channel estimation algorithm for ACDD is out of the scope of this paper, thereby we consider here a perfect channel knowledge for simplicity, but we expect that ACDD should suffer more than Alamouti under more realistic channel conditions.

III. Maximum-Ratio Combining Receive Diversity

In this section, we generalize the MISO models presented in the previous section to MIMO models. The MRC scheme can be applied by combining the received signals from different antennas in frequency domain. Consider the received signal in the n_rth receive antenna in frequency domain as

$$\mathbf{Y}_{n_\mathrm{r}} = \mathbf{\Lambda}_{n_\mathrm{r}} \mathbf{A}_{\mathrm{eq}} \mathbf{d}_{\mathrm{eq}} + \mathbf{W}_{n_\mathrm{r}}, \tag{12}$$

where $\mathbf{\Lambda}_{n_\mathrm{r}}$ is the equivalent SISO channel between the n_rth receive antenna and the transmit antennas used at transmitter, that can be obtained from the Alamouti, CDD or ACDD schemes. More precisely, equation (12) is replaced by one of the equations given in Table (II) according to the respective STC technique. Notice that the size of $\mathbf{Y}_{n_\mathrm{r}}$ depends on the STC technique, for instance, $\mathbf{Y}_{n_\mathrm{r}} \in \mathbb{C}^{2K_{\mathrm{on}}}$ for Alamouti and ACDD, while $\mathbf{Y}_{n_\mathrm{r}} \in \mathbb{C}^{K_{\mathrm{on}}}$ for CDD.

The combination of signals according to MRC is given by [9], [15]

$$\mathbf{Y} = \frac{\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{Y}_{n_\mathrm{r}}}{(\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{\Lambda}_{n_\mathrm{r}})^{\frac{1}{2}}}, \tag{13}$$

where the normalization is done for convenience such that the noise related term remains with distribution equal to its respective STC schemes, which is proven in the following by equation (15).

TABLE II
Linear Models for the STC Schemes to Replace Equation (12)

STC Technique	Equation
Alamouti	(3)
Cyclic Delay Diversity (CDD)	(8)
Alamouti w/ Cyclic Delay Diversity (ACDD)	(9)

Interestingly and conveniently, the single-input multiple-output (SIMO) model of (13) collapses into a SISO model given by

$$\mathbf{Y} = \mathbf{\Lambda} \mathbf{A}_{\mathrm{eq}} \mathbf{d}_{\mathrm{eq}} + \mathbf{W}, \tag{14}$$

where $\mathbf{\Lambda} = (\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{\Lambda}_{n_\mathrm{r}})^{\frac{1}{2}}$ is the equivalent channel matrix in frequency domain. The noise term has the same distribution is its respective STC system, i.e., since the noise elements of the different antennas independent and identically distributed, the covariance matrix of $\mathbf{W}$ is

$$\begin{aligned} \mathbb{E}(\mathbf{W}\mathbf{W}^{\mathrm{H}}) &\overset{(a)}{=} \frac{\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbb{E}(\mathbf{W}_{n_\mathrm{r}} \mathbf{W}_{n_\mathrm{r}}^{\mathrm{H}}) \mathbf{\Lambda}_{n_\mathrm{r}}}{(\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{\Lambda}_{n_\mathrm{r}})^{\frac{1}{2}} (\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{\Lambda}_{n_\mathrm{r}})^{\frac{1}{2}}} \\ &\overset{(b)}{=} \mathbb{E}(\mathbf{W}_1 \mathbf{W}_1^{\mathrm{H}}) \frac{\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{\Lambda}_{n_\mathrm{r}}}{\sum_{n_\mathrm{r}=1}^{N_\mathrm{r}} \mathbf{\Lambda}_{n_\mathrm{r}}^{\mathrm{H}} \mathbf{\Lambda}_{n_\mathrm{r}}} \\ &\overset{(c)}{=} \mathbb{E}(\mathbf{W}_1 \mathbf{W}_1^{\mathrm{H}}), \end{aligned} \tag{15}$$

where we obtain equality (b) due to $\mathbb{E}(\mathbf{W}_{n_\mathrm{r}} \mathbf{W}_{n_\mathrm{r}}^{\mathrm{H}})$ be diagonal and equal for all n_r, in which we selected $n_\mathrm{r} = 1$ without loss of generality. The modulation matrix $\mathbf{A}_{\mathrm{eq}}$ and the data vector $\mathbf{d}_{\mathrm{eq}}$ are given directly by the respective models of Table II.

IV. Data Estimation for OFDM and OCDM

We demonstrated that the transmit diversity techniques together with MRC are equivalent to a SISO system, given by equation (14). Therefore, signal processing in frequency-domain for estimating the transmitted data is straightforward. For instance, the transmitter matrix in frequency domain of OFDM is $\mathbf{A}_{\mathrm{OFDM}} = \mathbf{I}_{K_{\mathrm{on}}}$, therefore the estimation of data with any combination of STC technique with N_t transmit antennas and N_r receive antennas is straightforward, e.g., by means of linear minimum mean-squared error (LMMSE) estimation.

In this work, we analyze the deployment of the recently proposed OCDM [2] multicarrier system using the transmit diversity techniques. The transmitter matrix of OCDM is $\mathbf{A}_{\mathrm{OCDM}} = \mathbf{\Gamma} \mathbf{F}_{K_{\mathrm{on}}}$, being $\mathbf{\Gamma}$ a diagonal matrix with elements given by $\exp\left(-j\pi n^2/K_{\mathrm{on}}\right)$ for $n = 0, 1, \cdots, K_{\mathrm{on}} - 1$ [2]. Due to inter-symbol interferences, OCDM requires a more complicated type of receiver for estimating the data, therefore we will use the low-complexity receiver proposed in [3] which is directly applied in the frequency domain, since the model of equation (14) is equivalent to [3, eq. (2)].

V. Numerical Results and Comments

In order to analyze the performance of OFDM and OCDM waveforms for URLLC use cases, we consider a vehicular communication application based on the ITS-G5 standard that employs the transmit diversity schemes described in Section II, whose underlying PHY is derived from the 802.11 standard [12], [13]. The parameters are given in Table III. The channel taps are independent complex Gaussian random variables with power based on ITS-G5 channel model, which is given in Table IV [13]. For simplicity, we consider that the channel is invariant over two OFDM or OCDM symbols. Notice that in reality, the time-varying characteristic of the channel introduces additional inter-symbol interference and error in the channel estimation. However, this analysis is not in the scope of this paper. The codeword obtained from the $\{133, 171\}_8$ convolutional encoder is interleaved and spread over two OFDM or OCDM symbols, including the CDD STC technique, resulting in the low-latency transmission time of $16\mu s$. Finally, since we are interested in investigating transmit diversity, we consider $N_\mathrm{t} = \{1, 2, 4, 8\}$ antennas at transmission and $N_\mathrm{r} = \{1, 2\}$ receiver antennas. The outcomes for $N_\mathrm{r} = 1$ and $N_\mathrm{r} = 2$ are depicted in Fig. 3 and Fig. 4, respectively.

TABLE III
SIMULATION PARAMETERS

Parameter	Value
Available Subcarriers (FFT size)	64
Data Subcarriers	48
Bandwidth	10 MHz
OFDM/OCDM Block Duration	$6.4\mu s$
CP Duration	$1.6\mu s$
n. of OFDM/OCDM Blocks	2
Modulation and Coding	QPSK with 1/2 Code Rate
Encoder	$\{133, 171\}_8$ Convolutional
Number of iterations for OCDM [3]	5

TABLE IV
ITS-G5 HIGHWAY NLOS

	Tap 1	Tap 2	Tap 3	Tap 4
Delay $[ns]$	0	200	433	700
Power [dB]	0	-2	-5	-7

Looking first at Fig. 3, the primary observation to make is that OCDM outperforms OFDM in all scenarios. This is expected since OCDM spread the symbols equally in frequency [1], [3]. For the CDD technique (top graph in Fig. 3), the results show that the gap between OCDM and OFDM augments as N_t increases. This phenomenon happens because the equivalent SISO channel becomes more selective in frequency, favoring the spreading property of OCDM. On the other hand, the effect of Alamouti is to make the channel less selective, leading to two main differences in comparison to CDD. First, Alamouti[2] outperforms CDD with $N_\mathrm{t} = 2$. Secondly, the gap between OCDM and OFDM in the ACDD case starts to increase only for $N_\mathrm{t} > 2$, because only when N_t takes greater values than 2, the CDD schemes starts to influence.

Fig. 4 presents the results for $N_\mathrm{r} = 2$, where we define the SNR per receive antenna in order to provide a meaningful comparison to the case of $N_\mathrm{r} = 1$. Therefore, there are two main effects when N_r increases. First, the overall SNR at the receiver is increased for the same transmit power, since there is more signal being captured. Secondly, the channel becomes less selective in frequency similarly to the Alamouti case. Combining these two effects, the performance improvement compared with $N_\mathrm{r} = 1$ is very large in all situations. Comparing OCDM with OFDM, the effects are similar to the situation where $N_\mathrm{r} = 1$, however the performance gap is decreased considerably due to less selectivity of the equivalent channel. For instance, with $N_\mathrm{t} = 4$ in CDD case, the gap at FER of 10^{-5} for $N_\mathrm{r} = 1$ is 2 dB, whereas for $N_\mathrm{r} = 2$ the gap is decreased to 1 dB.

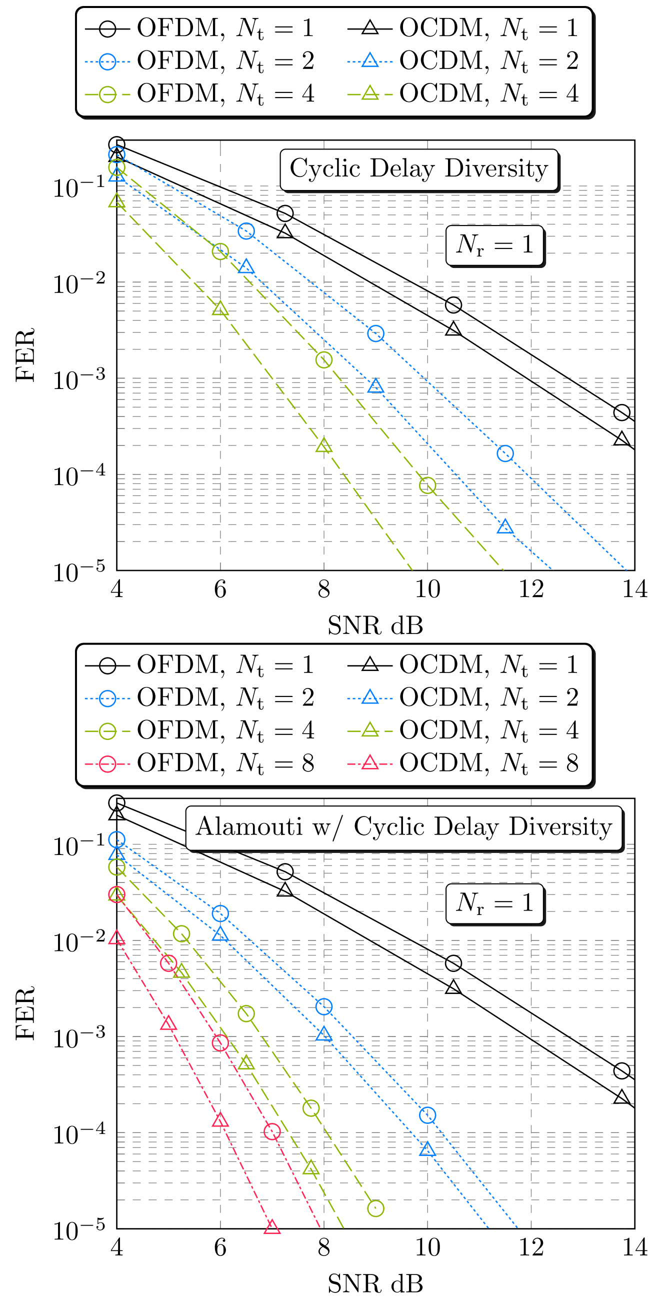

Fig. 3. FER results for CDD and ACDD schemes with $N_\mathrm{r} = 1$.

[2]Remember that Alamouti w/ CDD for $N_\mathrm{t} = 2$ is equivalent to Alamouti-only system.

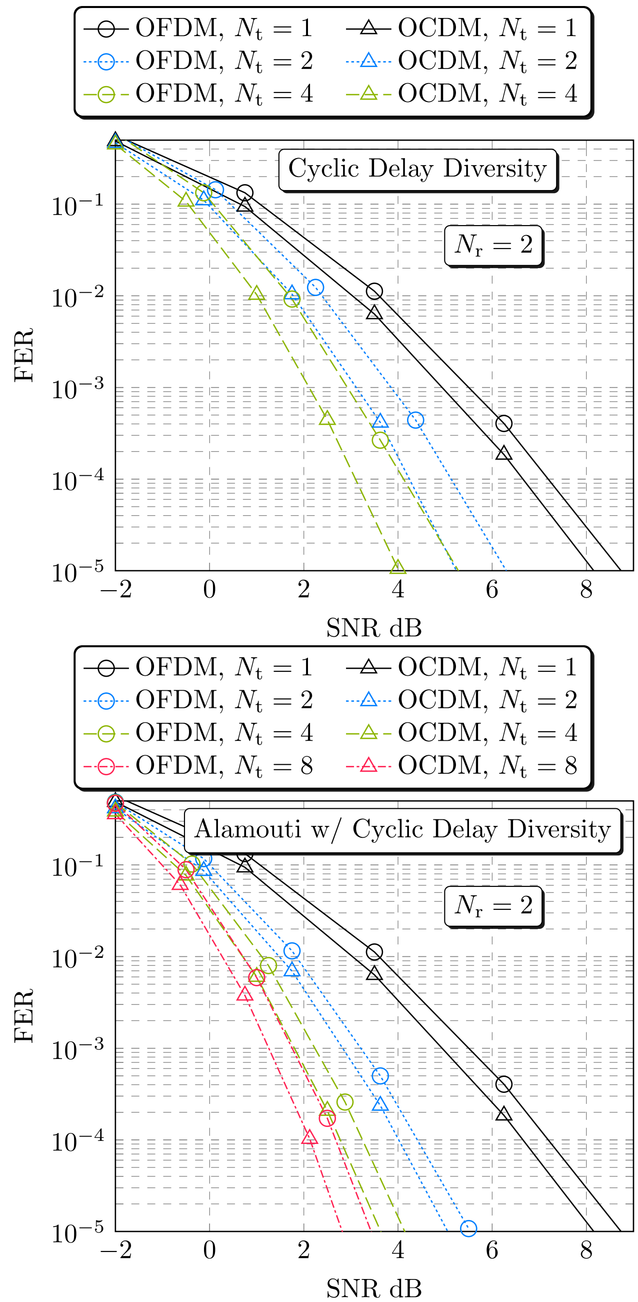

Fig. 4. FER results for CDD and ACDD schemes with $N_r = 2$.

It is worth highlighting that the presented results might be different under a more realistic scenario. For instance, the authors in [3] claim that the gap between OFDM and OCDM should increase when channel estimation is not perfect. In addition to that, we expect that imperfect channel estimation will have a greater impact in the curves for $N_r = 2$, since the channel mismatch is larger in a lower SNR region. Another important aspect is the channel estimation problem in the CDD technique. In this case, the channel estimation is more challenging due to extra selectivity of the equivalent channel. Therefore, in a more realistic scenario we expect that the gap between CDD and ACDD increases, making ACDD even more interesting in practice than CDD.

VI. Conclusion

In this paper, the performance of recently proposed orthogonal chirp division multiplexing (OCDM) and OFDM system have been compared in different MIMO space time coding schemes. As transmit diversity techniques, we have considered Alamouti, cyclic delay diversity (CDD) and Alamouti with CDD (ACDD). For the receive diversity we have used the well known maximum-ratio combining. We have first developed a detailed model in frequency domain for ACDD, which was not available in the literature. Then we demonstrated that OCDM outperforms OFDM for all the schemes, mainly due to the spreading property of OCDM. Regarding the transmit diversity techniques, our results revealed that CDD increases the gap between OFDM and OCDM, which happens due to extra channel selectivity. On the other hand, this gap decreases when the number of receive antennas increases. However, we highlighted that this effect may change when channel estimation is not perfect. Therefore, we suggest as future work the investigation of these STC schemes under a more realistic scenario, where we expect that OCDM waveform should suffer less than OFDM.

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